PALINDROMORPHY

Symmetries of the Keyboard

Their creative use in Improvisation & Composition

Victor Alcántara

ALCÁNTARA MUSIKVERLAG

© 2019 Victor Alcántara

Victor Alcántara (*1975) studied classical piano, jazz piano and film scoring at the Richard Strauss Conservatory Munich and the University of Music and Performing Arts Munich. He has performed as solo pianist, bandleader and sideman at festivals in Germany and abroad and has been engaged as pianist and composer for numerous radio and film productions. His teaching activities at German universities include jazz piano, piano improvisation, jazz arrangement and jazz composition. In 2017 he received his doctorate in musicology from the University of Music and Performing Arts Graz.

© 2019 Victor Alcántara

AMV-008-6
1. Edition 2019
www.amv-shop.de
ISBN 978-3-945722-03-9
English translation by Victor Alcántara

Cover graphic: Fotalia
Coverdesign: Rafael Alcántara, Victor Alcántara
© 2019 Alcántara Musikverlag, Munich
www.victoralcantara.de

All Rights Reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or in any means – by electronic, mechanical, photocopying, recording or otherwise – without prior written permission.

Printed in Germany

Foreword

Piano playing generally is characterized by a constant intermeshing of three of the five human senses: hearing, feeling and seeing. The sensory perception of the keyboard is at the same time an immediate experience of its topography. The systematic investigation of this bichromal surface structure is, in the broadest sense, the main subject of this book. It resembles a music-theoretical cartography based on symmetrical laws. Hereby especially the construction of scales is examined, above all their topographic representation on the keyboard.
In addition to numerous application examples, there are three original compositions for solo piano to be found at the end of the book. The extensive listing of scales that precedes them may have a structuring effect on further activities with composition, improvisation and analysis. At first glance, topographically influenced pianism may remain largely hidden from the audience: the sensory experience of the performers, however, allows the listener to experience a music that is deeply pianistic and therefore is in harmony with the instrument through which it speaks.

Victor Alcántara

© 2019 Victor Alcántara

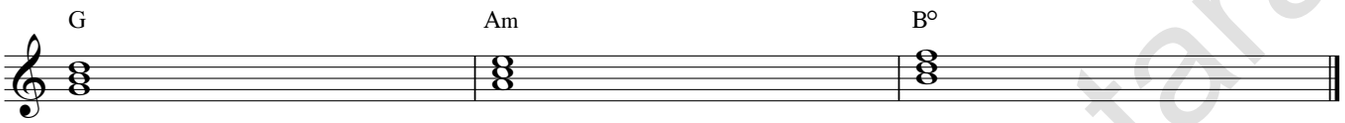
Content

© 2019 Victor Alcántara

© 2019 Victor Alcántara

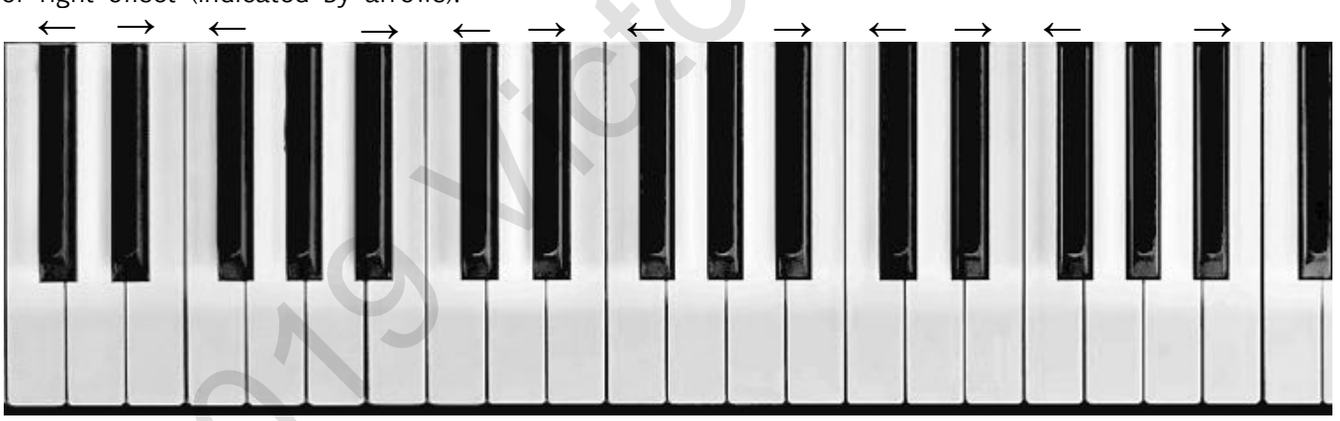
Phenomena of topography

The present shape of the keyboard is the result of centuries of structural adaptation, inter alia, to innovations with respect to music theory as well as to tuning systems of Western music. The sequence of the (black and white) keys is based on chromaticism: Nearest keys are arranged in semitone spacing. With regard to the surface design, there are 5 black keys and 7 white keys per octave – a topographical asymmetry that is a matter of course for pianists. The unbalanced ratio of black and white keys leads to ambivalences and has a noticeable effect e.g. on fingering as well as the transposition of musical structures. If, for example, only white keys are used, it can occur that certain topographical shapes are identical although they have a different interval structure. This is the case, for example, with a G major triad, an A minor triad and B° triad in root position. Accordingly, congruent topographical shapes may have different musical outcomes.



It is less obvious that this haptic mix-up also applies to certain combinations of white and black keys (or vice versa). Although the fingerings of the following chords are almost identical – the black key is not aligned exactly in the middle between the two white keys in chord 1 (Bm) and 3 (Bb/D) – the type of triad is completely different. In playing the chords as indicated below, it may be observed that the topographical shape of each chord is broadening minimally.[1]

The cause of the aforementioned phenomenon is shown in the following figure. Only the black G# key is aligned exactly between the two white G and A keys. The remaining black keys have a minimum left or right offset (indicated by arrows).



With respect to the pianist's haptical sensations, there may occur the following ambiguities: By the exclusive use of white keys, the minor seconds E/F and B/C may be mistaken for major seconds; the minor thirds D/F, E/G, A/C and Bb/D may be mistaken for major thirds; the tritone F/B may be be mistaken for fourths. This applies correspondingly to all their intervallic complements.
In combining white and black keys, the major thirds D/F#, E/G#, A/C# and B/D# may be mistaken for the perfect fourth F/Bb; the major thirds Ab/C, Db/F, Eb/G and Bb/D may be mistaken for the perfect fourth F#/B. This also applies to all their intervallic complements.
This topographical ambiguity is analyzed regarding the main intervals as follows: all types of intervals – with exception of the perfect unison[2] – may be built by pairs of white keys; the same applies to pairs of black keys, which additionally exclude semitones and tritones. Combinations of white and black keys may build all types of intervals except perfect unisons and octaves.

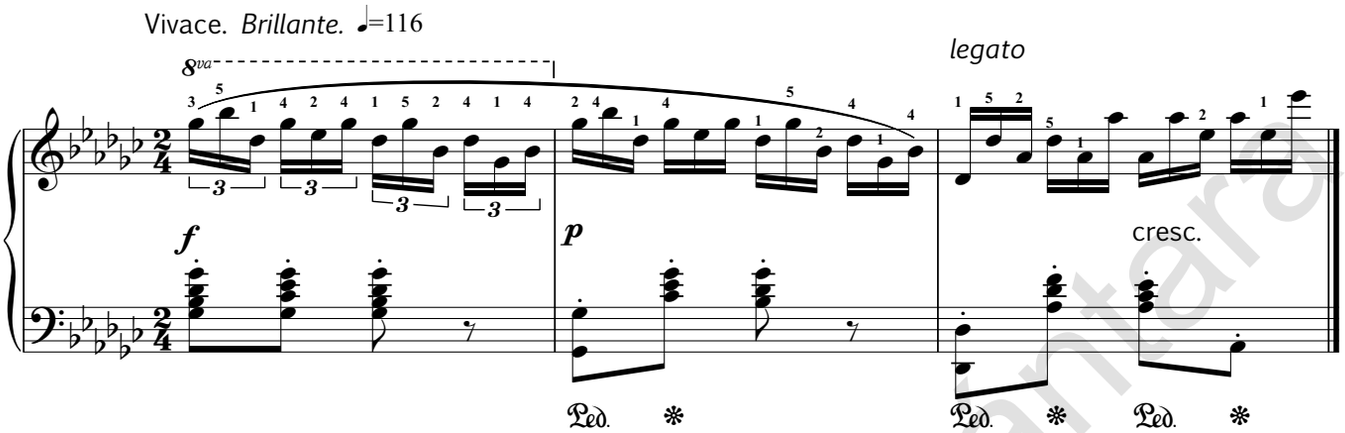
[1] If chords, intervals, and scales are notated diastematically (without rhythm), black keys will be represented by black noteheads, white keys by white noteheads.

[2] Of course, the idea of playing a perfect unison with two different keys seems strange at first, but: other chordal instruments, such as the guitar, can produce the same pitch on different strings.

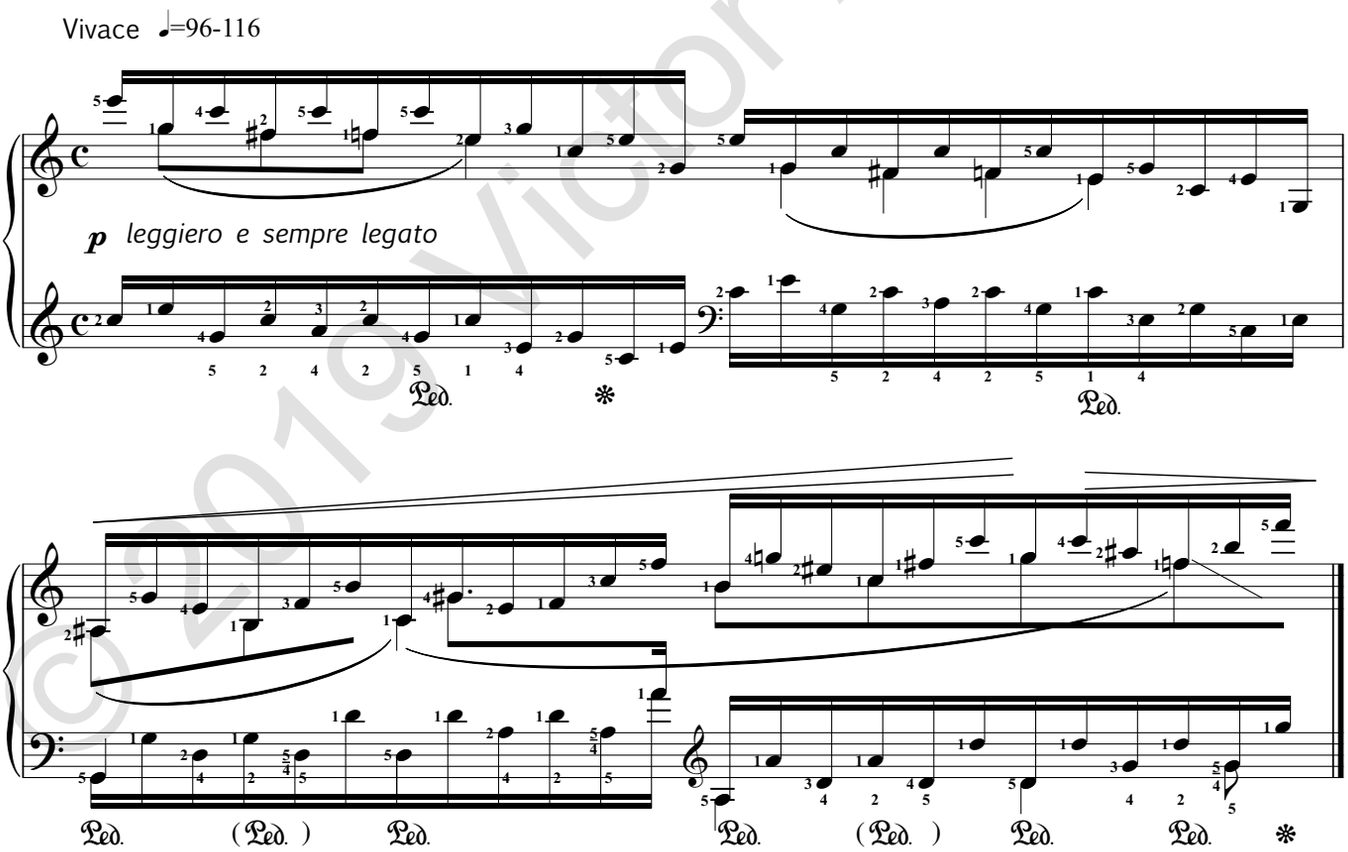
© 2019 Victor Alcántara

The topography of the keyboard as starting point of composing

Compositions that have a strong reference to the topography of the keyboard can be found in different styles of music history. A well-known example is Frédéric Chopin's (1810–1849) Etude Op. 10, no. 5 (composed in 1830, published in 1833): black keys are used in the right hand only. As follows, the first 3 bars of his composition.



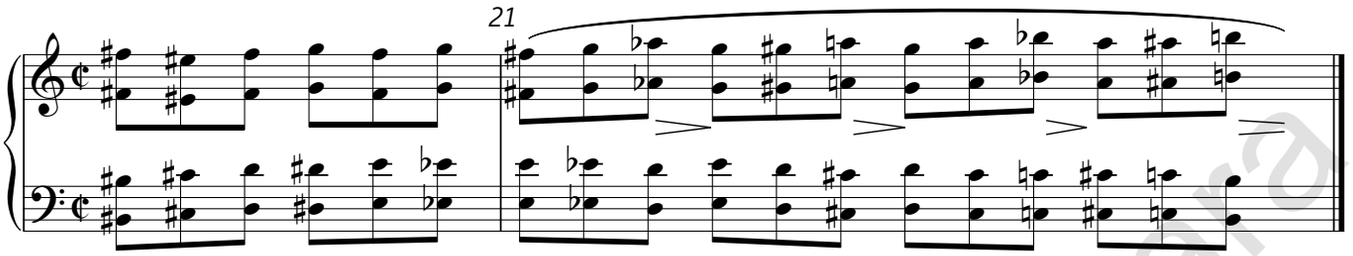
Leopold Godowsky (1870–1938) repeatedly edited the aforementioned piece in his *Studies on Chopin's Études* (composed 1894–1914). As to be seen from the opening bars of his 2nd variation (key: C major), the original assignments are interchanged: The left hand exclusively plays on white keys while the right hand uses white and black keys. In addition, the upper voice of Chopin's original is transferred to the lower voice.



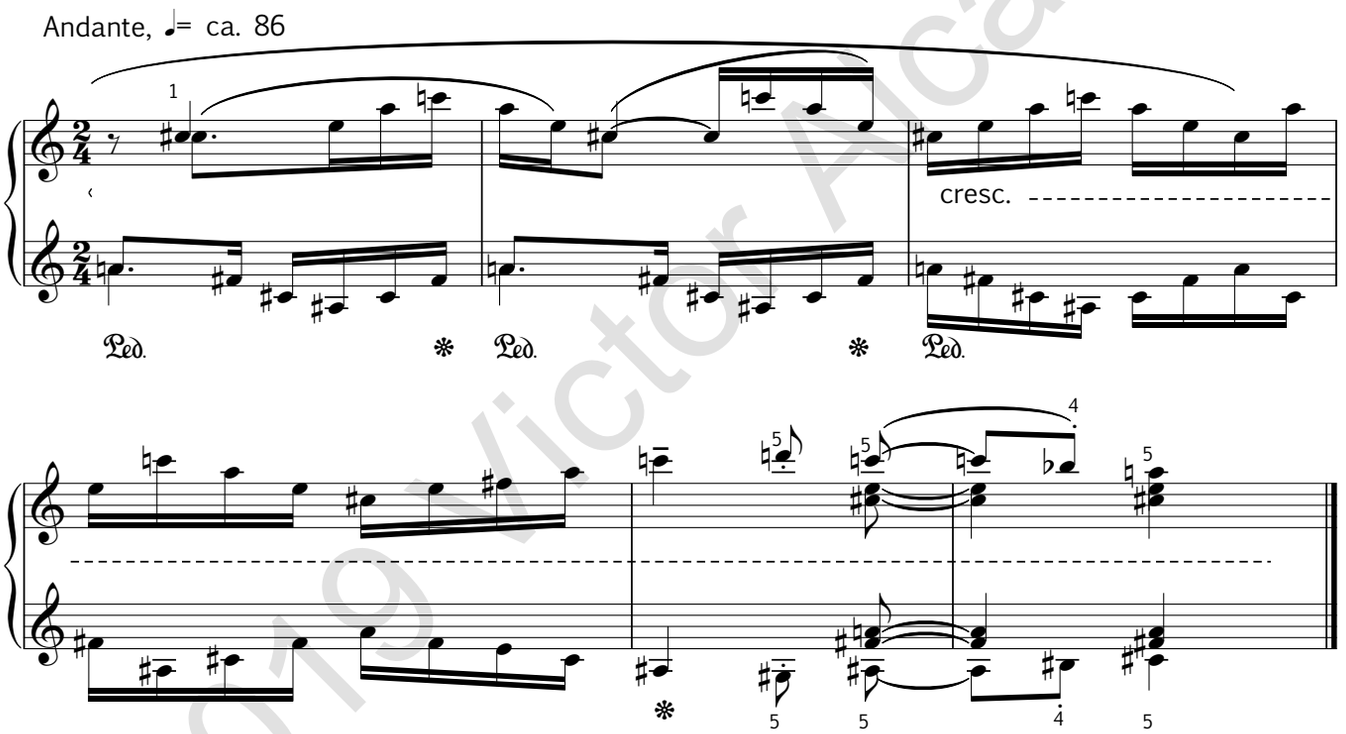
© 2019 Victor Alcantara

Topographical inversion in composition and improvisation

Topographical inversion is extremely rare to find in piano compositions, usually occurring only in fragments. From bar 20 (third beat) to the end of bar 21 of Study No. 10 (Op. 25) by Frédéric Chopin, the right hand consistently uses a white key when the left one uses a black key (or vice versa) – except for the last note in the following musical example. To the most part both hands run in chromatical contrary motion.



A presumably well-aimed implementation of the phenomenon of topographical inversion can be found in piece no. 143 "Divided Arpeggios" (Mikrokosmos, vol. 6, bar 18–23) by Béla Bartók (1881–1945). The inversion proceeds mirror-symmetrically and occurs either metrically offset or ambidextrous-synchronously



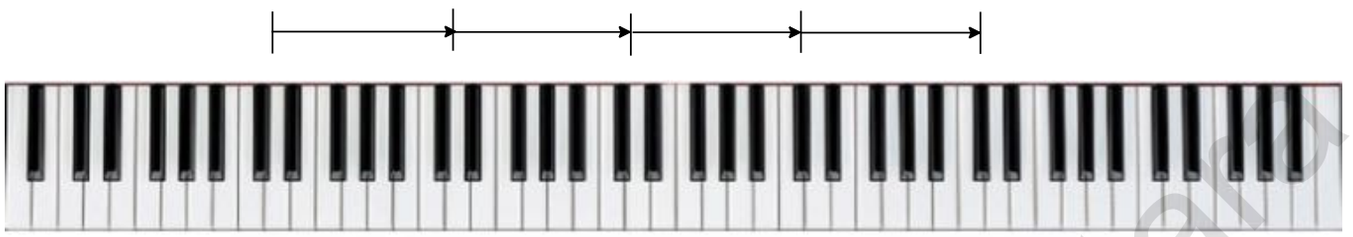
David Rakwoski's (*1958) etude "Narcissitude" is also determined by topographical inversion (David Rakowski, 2007, Études, Book 8, Peters, New York, 2007). In jazz, such techniques can also be found, as in Eldar Djangirov's improvisation to "Point of view redux" (section 01'31'' to 01'59'' in: Eldar Djangirov, 2013, *Breakthrough*, CD, Motema 233714) or in Kevin Hay's improvisation to "The Dervish of Harlem III" (section 02'35 to 03'04'' in: Kevin Hays, 2011, *Variations*, CD, Pirouette, PIT 3059). However, all in all, the systematic use of topographical inversion in composition and improvisation must be described as negligible.

The topographical phenomena discussed so far are based on two forms of symmetry. These will be discussed in the following and will be examined section by section in the course of the book under various aspects.

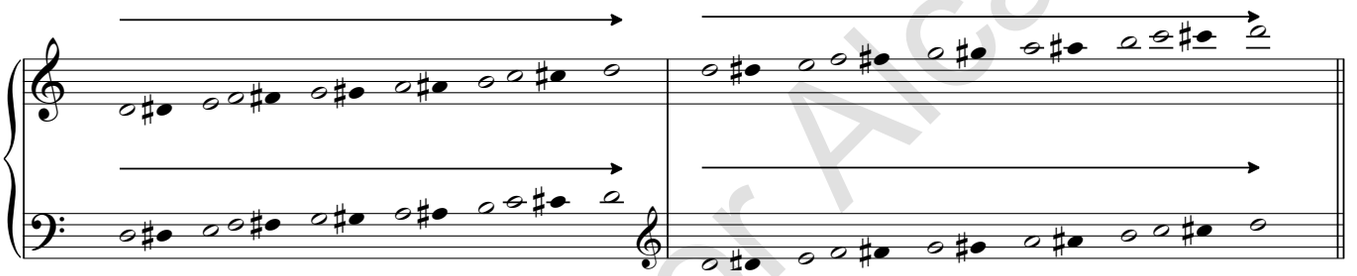
© 2019 Victor Alcantara

I. The translational symmetry of the keyboard

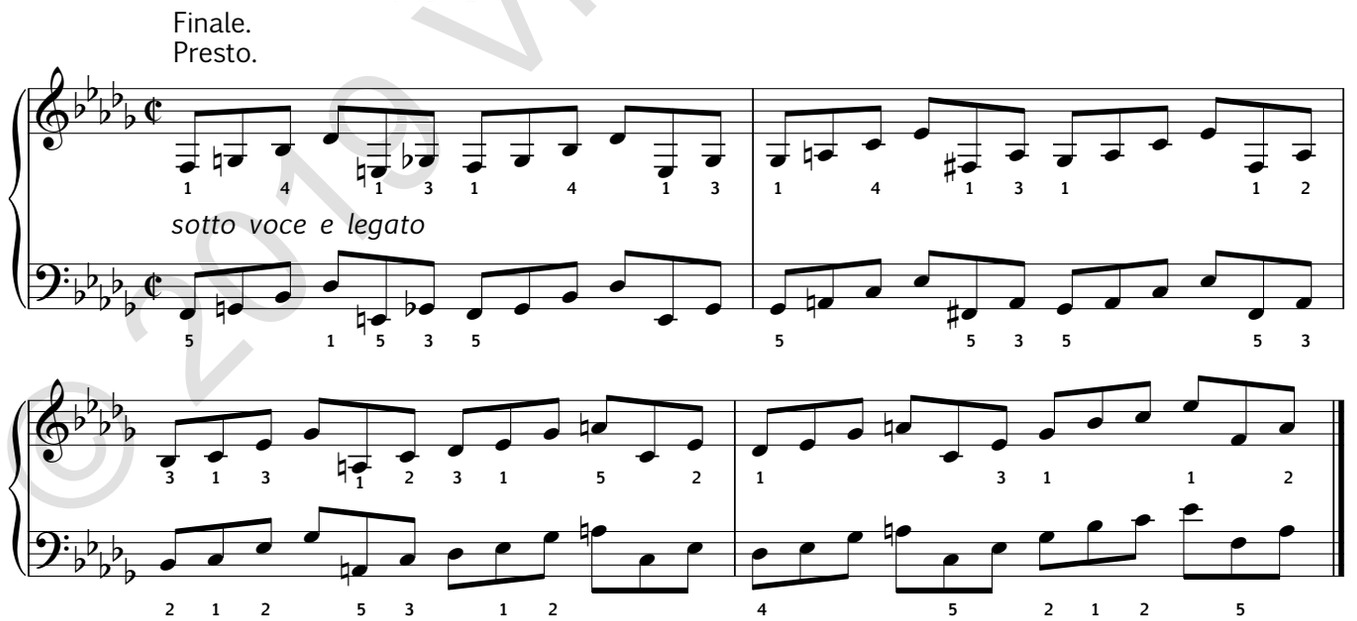
The keyboard is structured by translational symmetry. This means that the arrangement of the keys is repeated periodically every 12 semitones, regardless of which black or white key serves as the starting point of the continuous pattern. Translational symmetry is always given when objects remain the same as they are shifted. Each of the following arrows marks the range of an octave. The visual pattern repeats itself four times within the selected section (between D_2 and D_6).



From a musical point of view, the translational symmetry of the keyboard shown above could be realized, for example, by playing with both hands a chromatic scale in parallel motion, whereby the fingering of each hand would turn out differently. Translational symmetry is re-examined from a scalar perspective in the chapter "Scales" (p. 16).



The parallel motion of the hands usually is trained from the very beginning when learning to play the piano. Frédéric Chopin dedicates the entire final movement of his Sonata in Bb minor, op. 35, composed in 1839/1840, to this kind of playing.

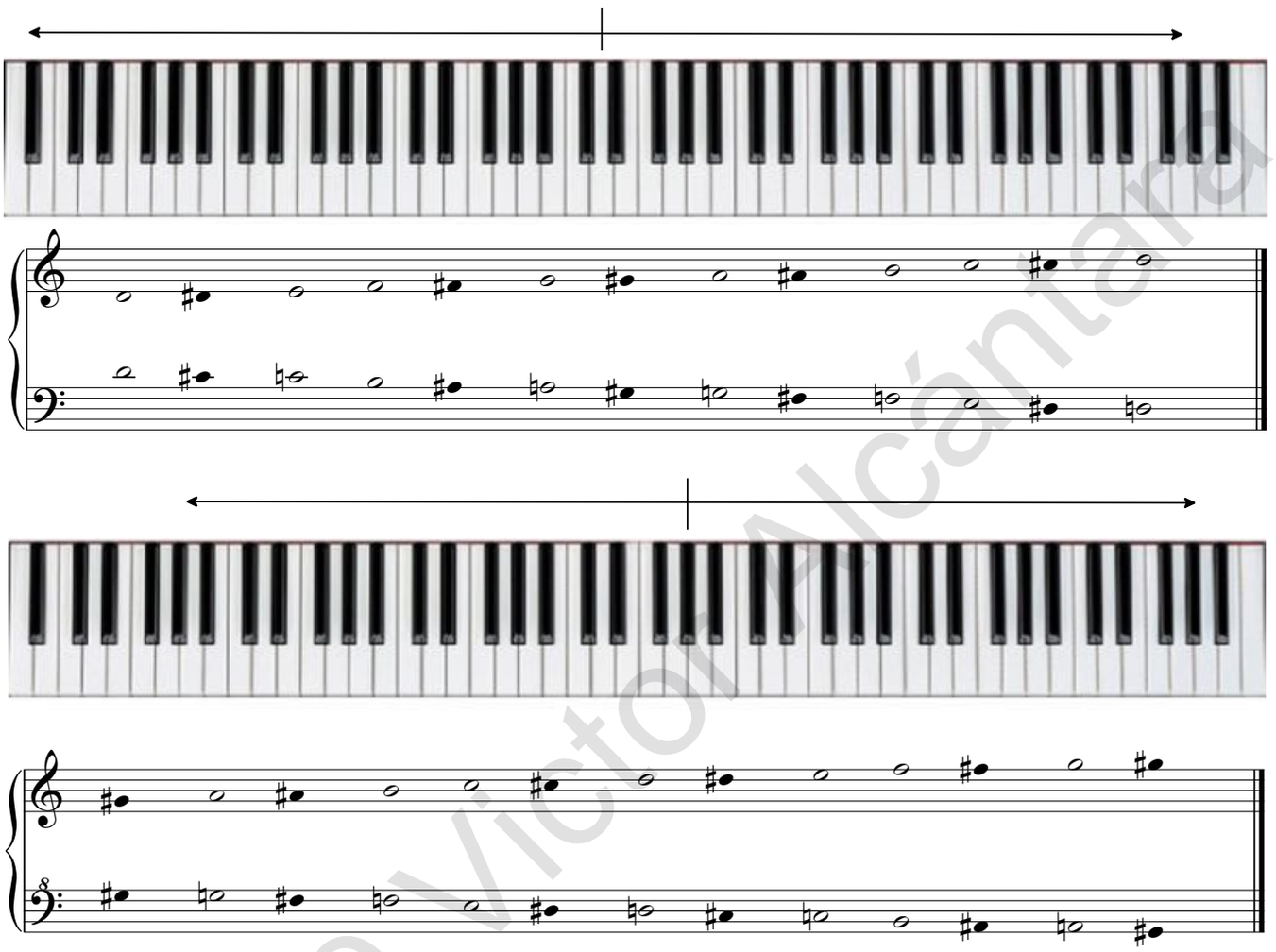


The above fingering was given by the composer – it is apparent, both hands have to use diverging sequences of finger combinations in parallel play. This core principle of translational symmetry with regard to pianism is a crucial difference to that other form of symmetry that also determines the keyboard layout and is the focal point of this book.

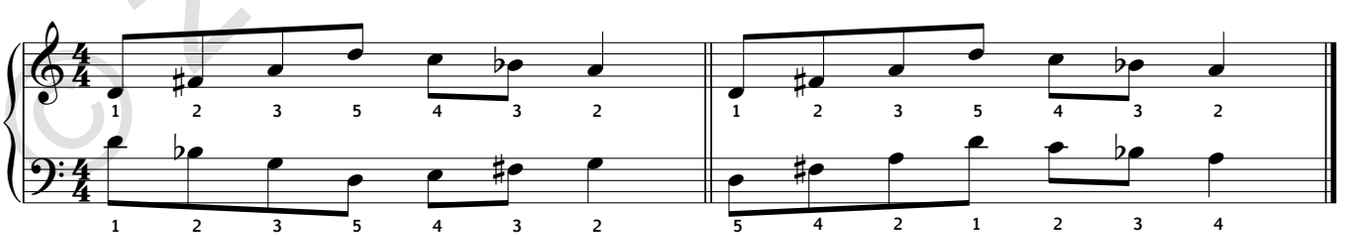
© 2019 Victor Alcántara

II. The mirror symmetry of the keyboard

The layout of the keyboard is mirror-symmetrical with keys D_4 and $G\#_4$ (or Ab_4) acting as topographical mirror axes. Aside from translational symmetry, mirror symmetry is another ordering principle of the keyboard. By playing a chromatic scale with the initial tone d or g sharp in contrary motion, the same sequence of black and white keys results in both hands.



The mirror symmetry of the keyboard, in contrast to the translational symmetry also present, is consistent with the human physiognomy; it therefore enables identical movements to be performed with both hands in terms of gross and fine motor skills. With respect to counterpoint, mirror symmetry is represented by contrary motion, the aforementioned translational symmetry by parallel motion, which is illustrated by the following comparison.

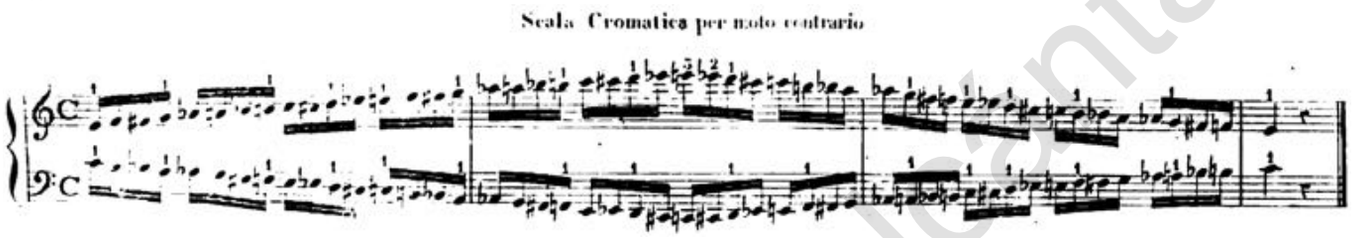


The above example shows once again that both identical fingerings and identical keys (black/white) can be used in playing mirror-symmetrically. However, in bar 1 it can also be seen that the notes are different on both sides, which on the whole may be perceived as sounding dissonant. Before an investigation of mirror symmetry with regard to tonal phenomena is made – related questions relating to improvisation and composition guide almost the entirety of this book – an investigation of the historical occurrence of topographical mirror symmetry in piano didactics is undertaken.

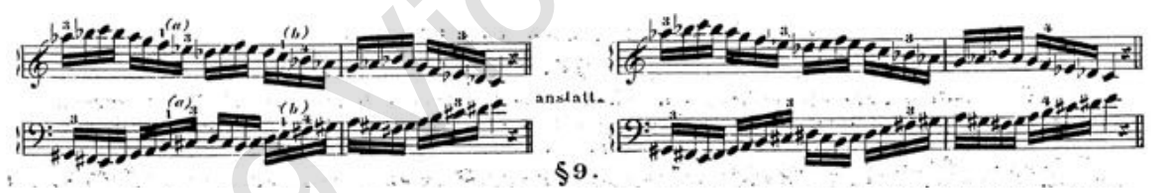
(c) 2019 Victor Alcántara

Mirror-symmetrical training in piano didactics

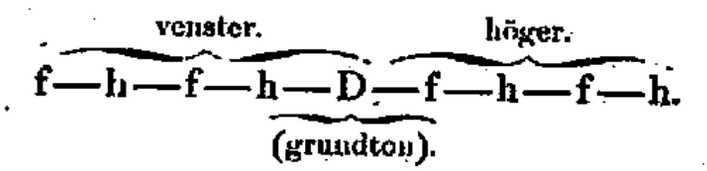
The history of topographical mirror symmetry as a means of piano didactics has been relatively unexplored so far. Bernhard Ziehn's (1845–1912) *System der Übungen* (1881, Hamburg: Pohle) is considered an extensive work of mirror-symmetrical etudes. His theory of symmetrical inversion, which emerged from this, influenced personalities such as Wilhelm Middelschulte (1863–1943) and Ferruccio Busoni (1866–1924). Carl Eschmann-Dumur's (1835–1913) *Guide de jeune pianiste* (1888, Zurich: Hug frères) was published 7 years after Ziehn's book and is considered for the first time to be a profound examination of mirror-symmetrical fingerings in the study of scales. The following sources are not mentioned in the sparse publications on the subject.
Francesco Pollini (1762–1846), pianist and composer of Italian origin, depicts the mirroring of a chromatic scale in his textbook *Metodo per il clavicembalo* (Giovanni Riccordi, Milan, p. 30) published in 1811. This was most likely a practice already widespread before the publication of the book. Pollini does not comment further on the phenomenon of mirror symmetry in this publication, the first Italian piano school.



The Austrian pianist and composer Johann Nepomuk Hummel (1778–1837) attracted the attention of his contemporaries early on and, as a young eleve, was at times even accepted into the household of Wolfgang Amadeus Mozart (1756–1791). It is therefore not odd to assume that Hummel also passed on the knowledge of his famous teacher in his very extensive textbook *Anweisung zum Piano-Forte-Spiel* (Tobias Haslinger, Vienna, p. 169), published in 1828. Although it contains a few examples of bilateral-symmetrical exercises, there is no explanation regarding the mirror symmetry of the keyboard.



Carl Erik Södlings (1819–1884) *Nytt system för piano-undervisning* (1860, Stockholm: Beckman) is probably the first manual of its time in which the training of the hands is based exclusively on mirror symmetry. A letter of recommendation by Sigismund Thalberg (1812–1871) from 1856 printed at the end of the booklet suggests that he had already taught according to his system four years before going to press. Södling, who worked as a music/sports teacher, folk song researcher and organist in Sweden and even taught in Buenos Aires as an exile at times, does not illustrate his system in music notation but by note names: The so-called "grundton" D serves as mirror axis of the pitches of the left ("venster") and of the right ("höger") hand.



Probably the first German-language publication on the methodology and didactics of mirror-symmetrical pianism is to be attributed to Carl Dorius Fuchs (1838–1922), a former student of Hans von Bülow (1830–1894), friend of Friedrich Nietzsche (1844–1900) and later patron of Hugo Riemann (1849–1919). Fuchs's five-part essay published in 1869 „Der Grundsatz der technischen Identität symmetrischer

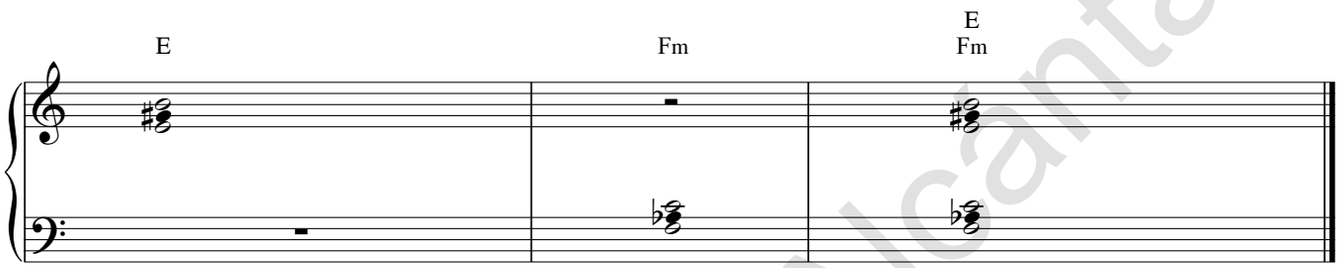
© 2019 Victor Alcantara

Horizontal mirroring of chords and scales

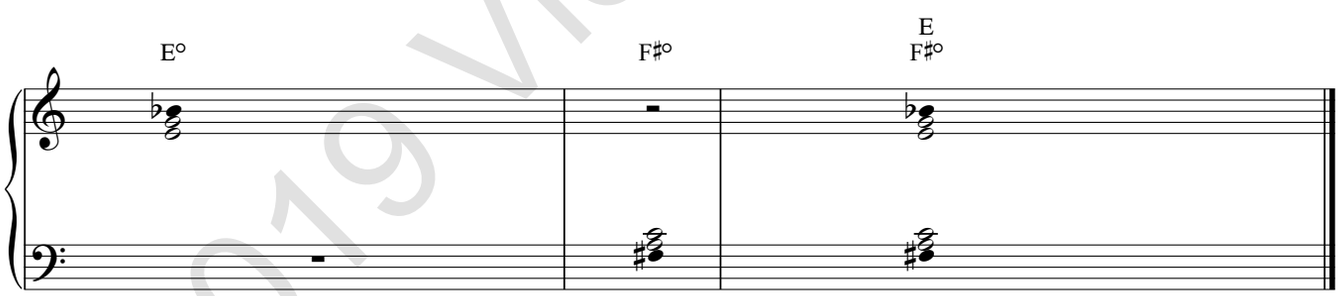
Since chords and scales belong to the basic elements of music, in the following special features are mentioned in relation to their horizontal mirroring. For reasons of clarity, a certain selection of triads and commonly used scales is examined.

I. Chords

A fundamental observation in the section "Considerations regarding functional harmony" (p. 10) was that the reflection of a major triad results in a minor triad (or vice versa) – the interval structure is reversed. This finding is as elementary as it is far-reaching: the overall impression of compositions based on thirds changes drastically by mirroring. If, for example, an E major triad (bar 1) is mirrored in root position along the axis D_4, the result is an F minor triad (bar 2) whereby both hands use the same keys If the chords are played at the same time (bar 3), the resulting sound is dissonant.

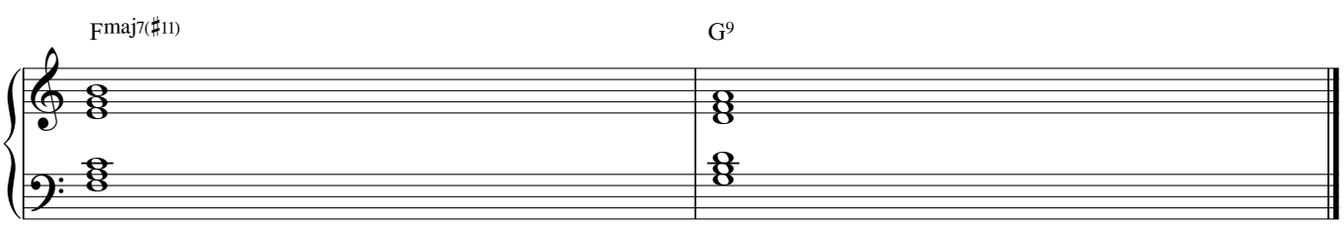


Triads whose interval structure is symmetric automatically produce the same structure when mirrored. In the following, this is shown by means of a diminished triad, which consists of two minor thirds. When mirrored around the axis D_4 – the triad again is diminished, but with different root notes. E° (bar 1) becomes F#° (bar 2), combining both chords results in yet another dissonant sound (bar 3).



Dissonances, however, are not an inevitable consequence of mirroring. A horizontally mirrored E minor triad becomes an F major triad, altogether resulting in an $Fmaj^{\#11}$ chord (bar 1); a horizontally mirrored D minor triad becomes a G major triad, altogether resulting in G^9 (bar 2)

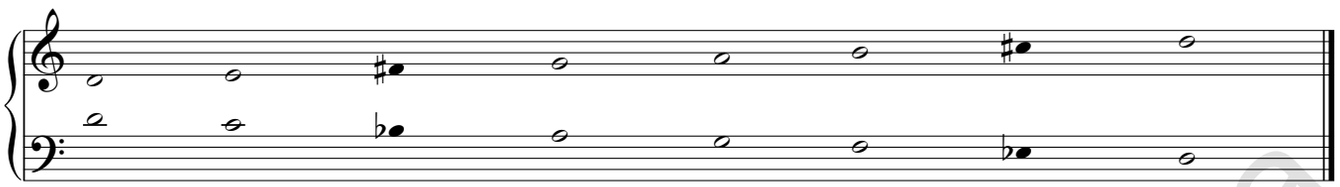
.



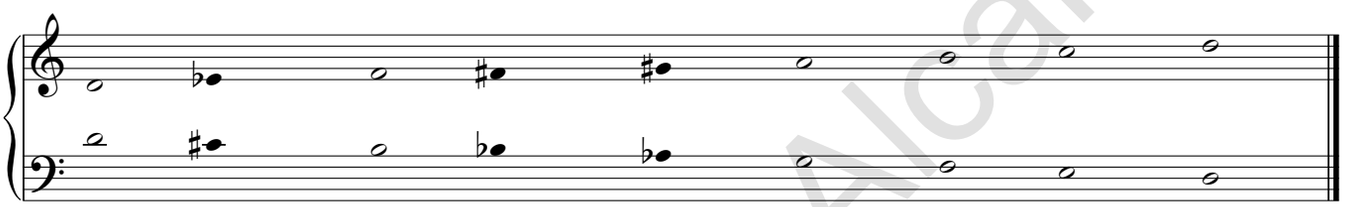
© 2019 Victor Alcantara

II. Scales

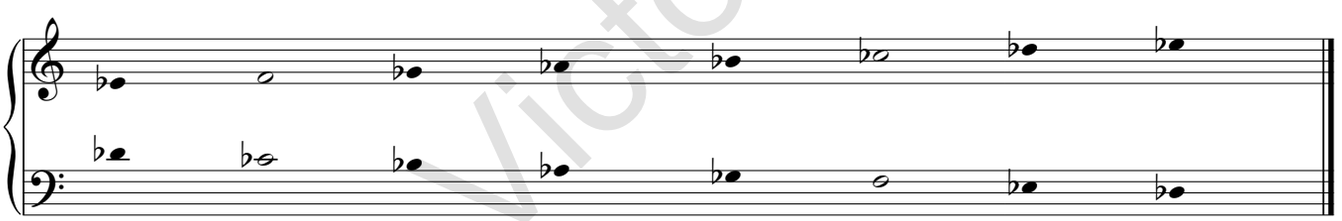
If one mirrors a D major scale on the D_4, one gets a Bb major scale: D Ionian becomes D Phrygian. The core principle is: 2 # become 2 b, 3 # become 3 b etc., the notes of both scales are not identical.



If one mirrors a symmetrical half-tone-whole-tone scale around the D, one automatically gets a symmetrical scale again, whereby the interval sequence – counted from the bottom to the top – forms a whole-tone-half-tone scale. The notes of both scales are identical in 4 of 8 cases.



However, bitonality is not an inevitable consequence. An Eb minor scale mirrored around the axis D_4 becomes an Eb flat minor scale again.



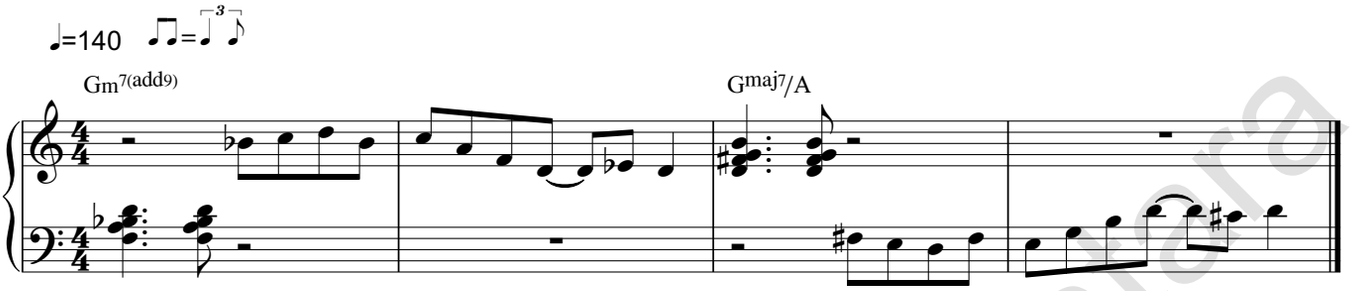
The question arises as to what are the mechanisms that determine whether there occurs either bitonality or unitonality by mirroring scales. For a better understanding of the music-theoretical correlations, the phenomenon "scale" itself is examined first, especially from a symmetrical point of view. The subsequent definition of a new term is aiming at the creative use of the knowledge gained.

© 2019 Victor Alcántara

Below are two simple examples of mirror-symmetrical piano playing that could be realized in a jazz trio improvisation. Typically, the pianist's left hand plays the corresponding chords, accompanied by a walking bassline in the bass and a swing groove in the drums.

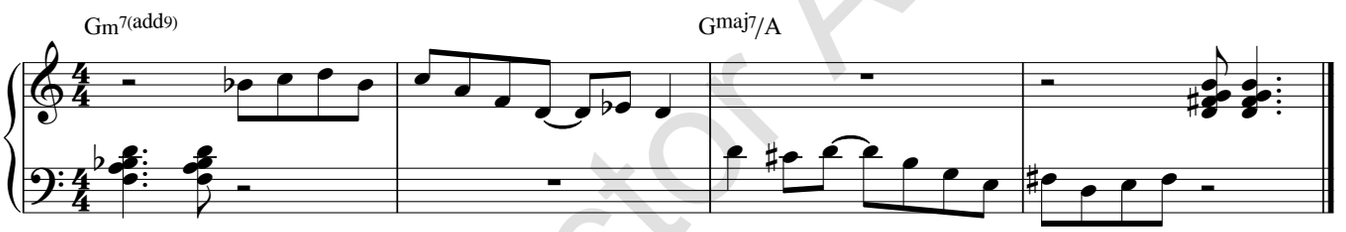
Horizontal Mirroring

In the following example bar 3 and 4 is nothing other than a plain horizontal mirroring of the musical passage of the first two bars.



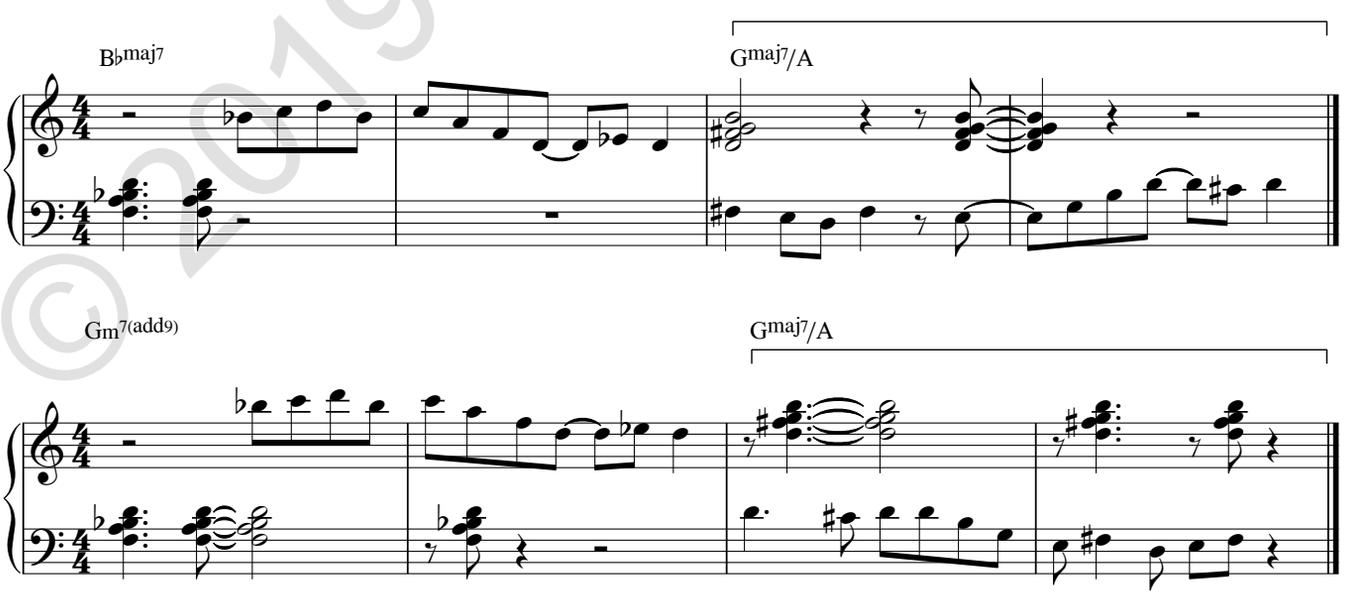
Combination of vertical mirroring with horizontal mirroring

Bar 3 and 4 show a combination of vertical with horizontal mirroring: The first two bars are played backwards in the following two bars, whereby the simultaneous horizontal mirroring around the D ensures that both hands use the same order of black and white keys.



Broken symmetry concerning rhythm

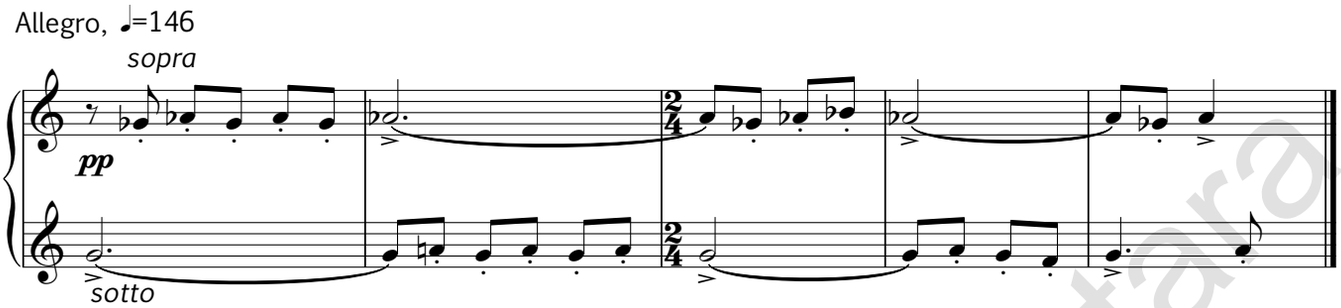
Concerning rhythm, broken symmetry may enrich the previous examples (see square brackets in the following example); the result is more variety.



© 2019 Victor Alcántara

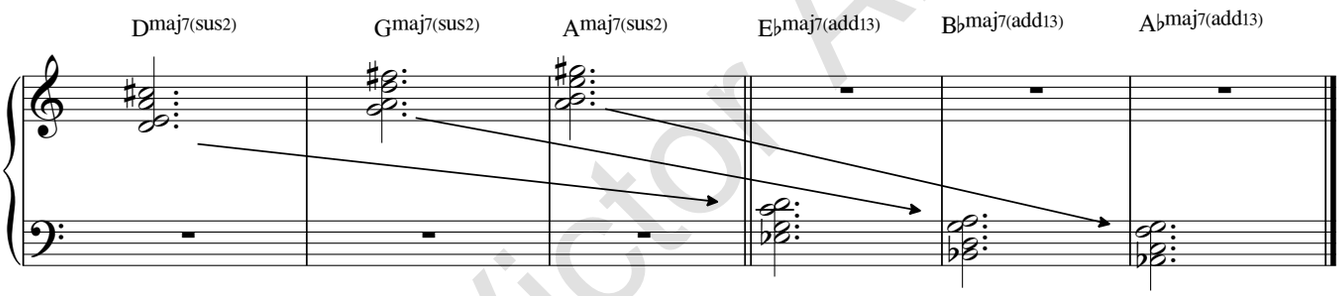
Horizontal and vertical mirroring combined with topographical inversion

Horizontal and vertical mirroring combined with topographical inversion is a very special approach to piano composition. Béla Bartók (1881–1945) made use of this technique in the first 5 bars of his piece No. 142, "From the Diary of a Fly" (Mikrokosmos, Vol. 6). The right and left hand continuously play in opposite directions, with every second bar corresponding to the preceding bar of the other hand.

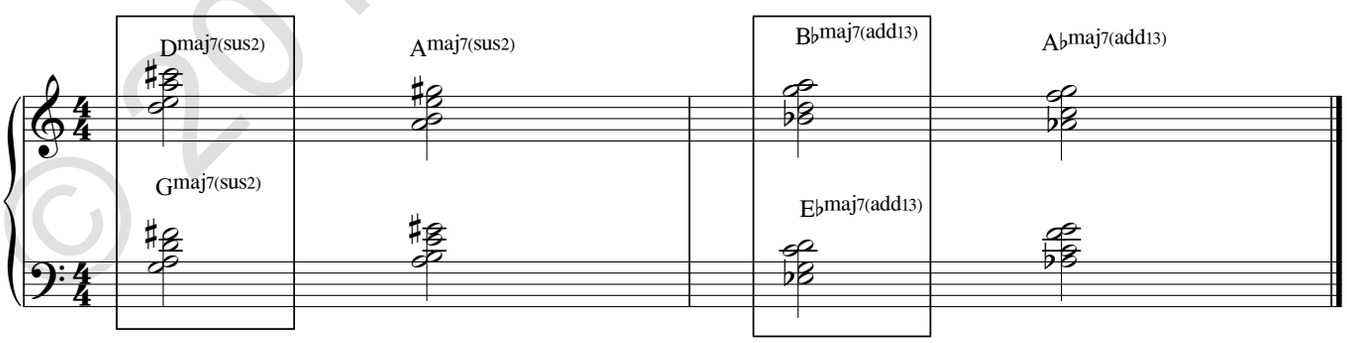


An approach using chords

Using a multi-stage procedure, the following illustrates how an individual compositional implementation of the aforementioned aspects may look like. As shown below, the same (intervalllic) combination of 3 white keys and 1 black key per voicing can be created three times (bar 1–3: $D^{maj7(sus2)}$, $G^{maj7(sus2)}$, $A^{maj7(sus2)}$). This applies accordingly to their mirroring (bars 4–6: $Eb^{maj7add13}$, $Bb^{maj7add13}$, $Abmaj7^{add13}$) around the D.

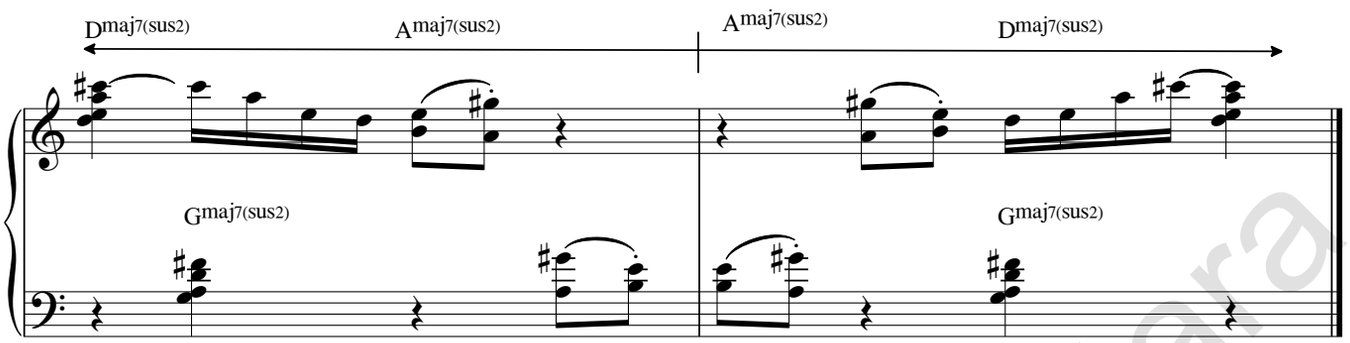


The chords of the first two bars of the above example are now assigned to one hand and played simultaneously, which is indicated by boxes; the chord of the third bar, on the other hand, instead is played octavated by both hands. The same procedure applies to the mirroring (bars 4–6 of the above example) in bar 2.

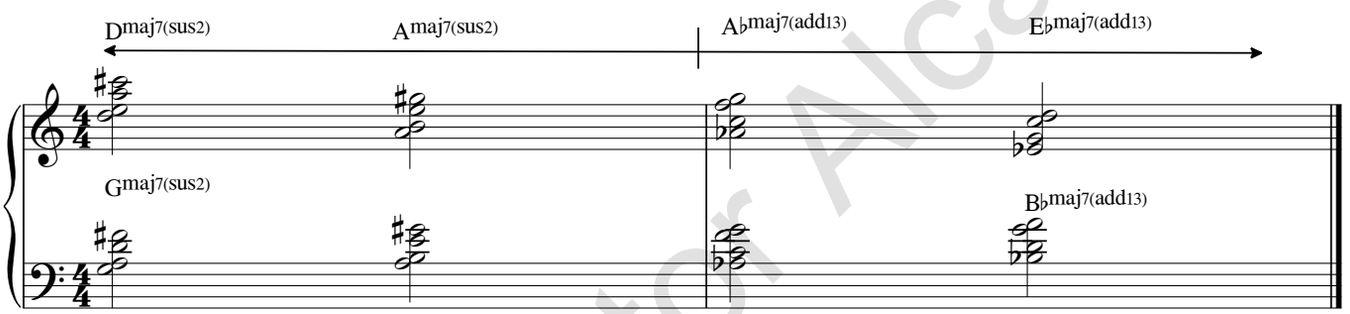


© 2019 Victor Arántara

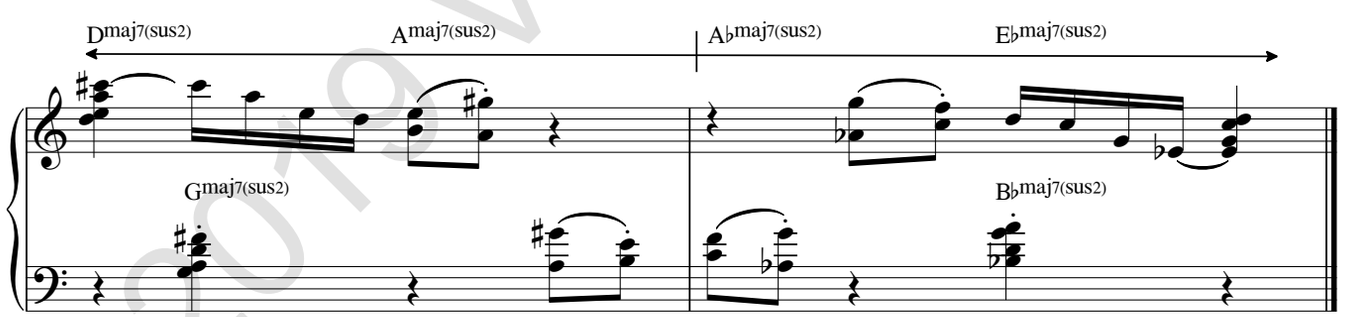
Bar 1 first shows a compositional implementation of the first two chords of the previous example; the voicings are played rhythmized. In Bar 2 the structure of bar 1 is played backwards respectively vertically mirrored.



The vertical mirroring, however, could also be combined with topographic mirroring. In the first place the chords are listed, i.e. bar 1 is topographically inverted in bar 2, as shown below.



Applying rhythm to the vertical inversion combined with topographic inversion may look as follows.

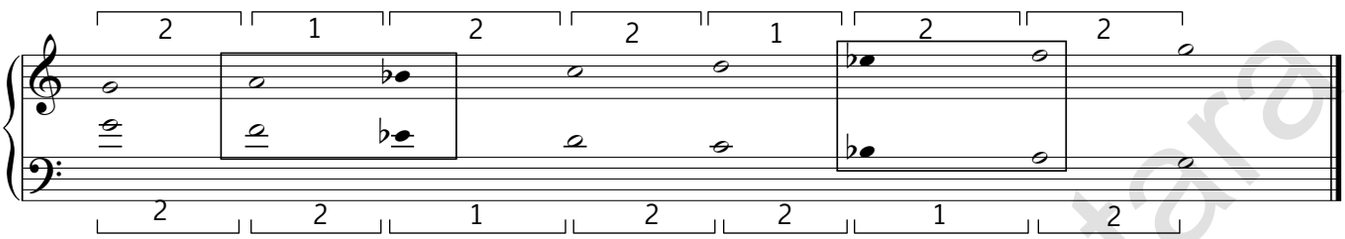


Finally, the second bar of the previous example is interchanged in the left and right hand. In other words: The right hand of the following example plays in measure 2 the vertical mirroring of measure 1 of the left hand with an additional horizontal mirroring, which is indicated by frames. The left hand proceeds analogously, the tonal result is harmonious overall.

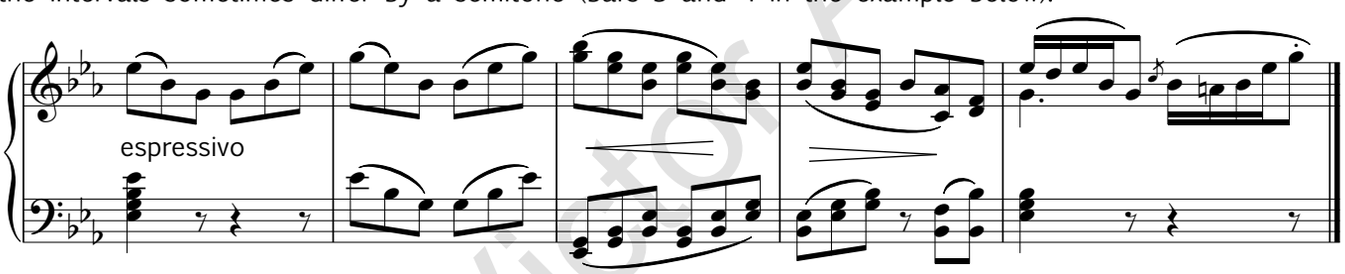
© 2019 Victor Alcantara

Palindromorphoid scales – broken symmetry

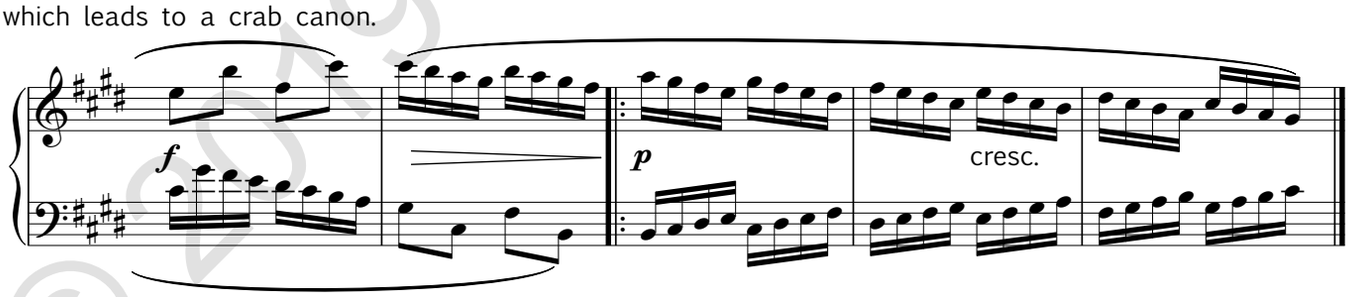
In addition to palindromorphic structures (intervals, chords, scales), there are numerous other topographic phenomena that may be described as mirror-like and which in the following are referred to as *palindromorphoid*. An example of a palindromorphoid scale is G minor (Aeolian): when playing it in contrary motion starting from the G, one will always find the same black or white key in both hands at the same time, despite the fact that the interval order differs by a semitone (see boxes) in several places when playing up or down.



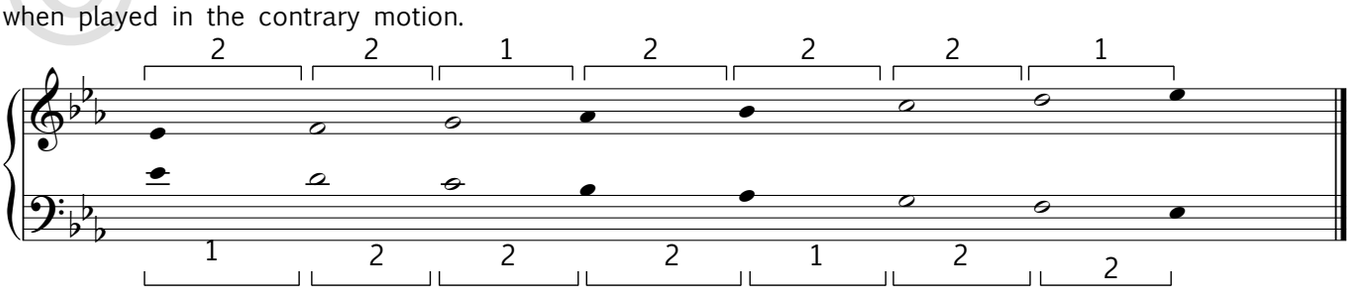
Palindromorphoid structures differ from palindromorphic scales in that, when playing in the opposite direction, a semitone difference may occur several times, although the order of black and white keys and the pitches are identical with both hands. For example a palindromorphoid structure such as an E flat major triad can be extracted from the G minor scale, and then played with both hands in mirror-like contrary motion. An example of this can be found in bar 7–8 of the 3rd movement of Beethoven's sonata "Les Adieux", Op. 81a: The same white or black keys are found in both hands, while the intervals sometimes differ by a semitone (bars 3 and 4 in the example below).



The piano work Muzio Clementis (1752–1832) also contains palindromorphoid structures. In piece no. 73 of his piano textbook "Gradus Ad Parnassum" (composed around 1817–1826) both hands play the same types of black and white keys in contrary motion in bars 12–14 (bars 3–5 in the example below). They are extracted from an E major scale. In addition, the left hand mirrors the right hand metrically offset, which leads to a crab canon.



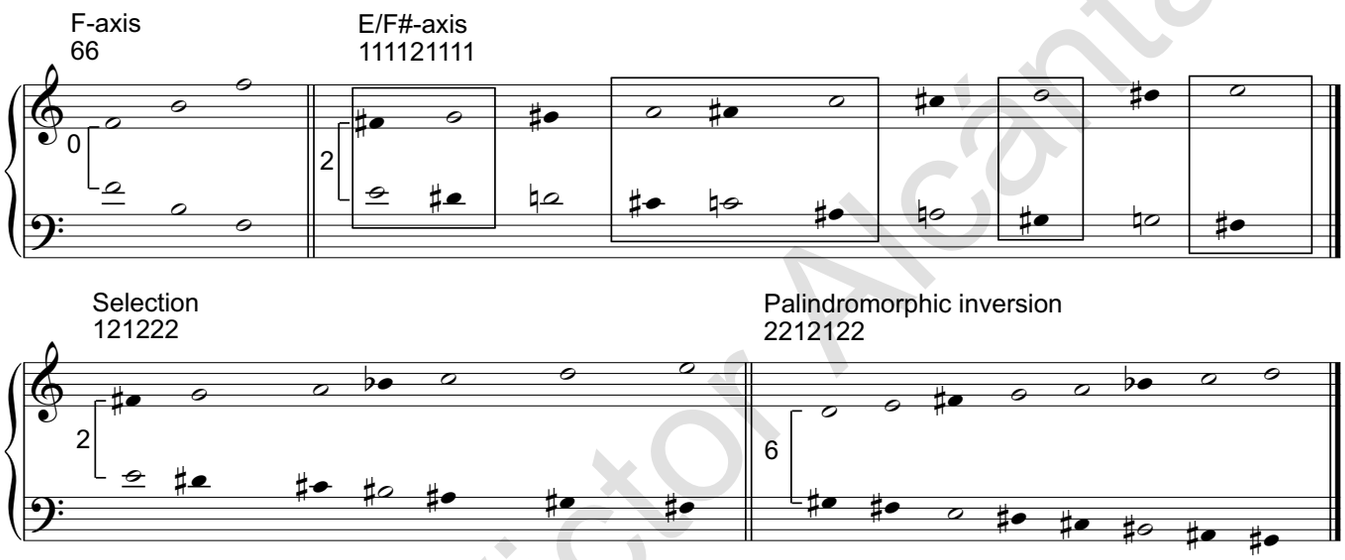
Examining a scale in Eb major from a topographical point of view, it can be seen that it is mirror-like when played in the contrary motion.



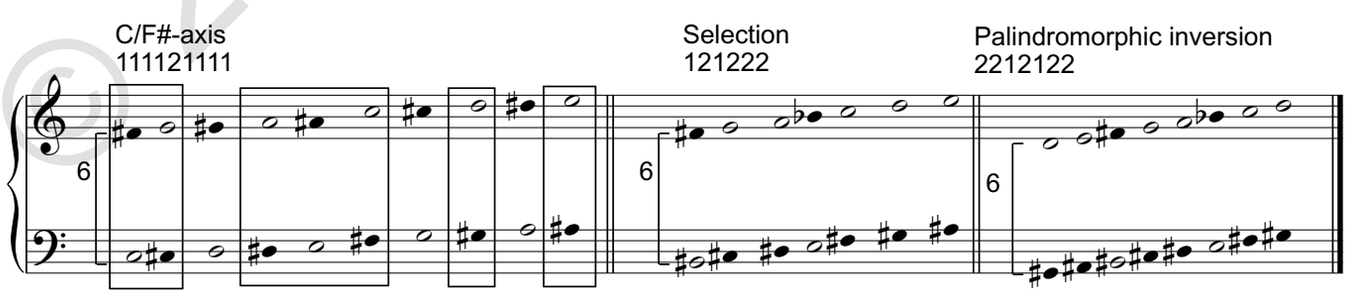
© 2019 Victor Alcantara

Complements with respect to topographical inversion

The listing of scale combinations that have been presented so far provide an opportunity to analytically get to the bottom of the phenomenon of topographically inverted playing. It will be shown that the topographical inversion is based on complements of palindromorphic primary scales. In the following example, this is shown by means of the 5th mode of G melodic minor – a palindromorphic scale – in contrary motion.
In the 1st staff the 1st bar shows the palindromorphic primary scale of the F axis, the 2nd bar its complement marked with frames. In the 2nd staff the 1st bar extracts the framed notes of the complement with enharmonic shift of the notes A# to Bb and C to B#. This results in G melodic minor in the right hand and C# melodic minor in the left hand. In the 2nd bar, the scales begin with the notes D (r. h.) and G# (l. h.), resulting in a mirror-symmetrical interval structure. It can be seen that the topographically inverted playing of the palindromorphic scale 2212122 (contrary motion) is related to the E/F# axis.



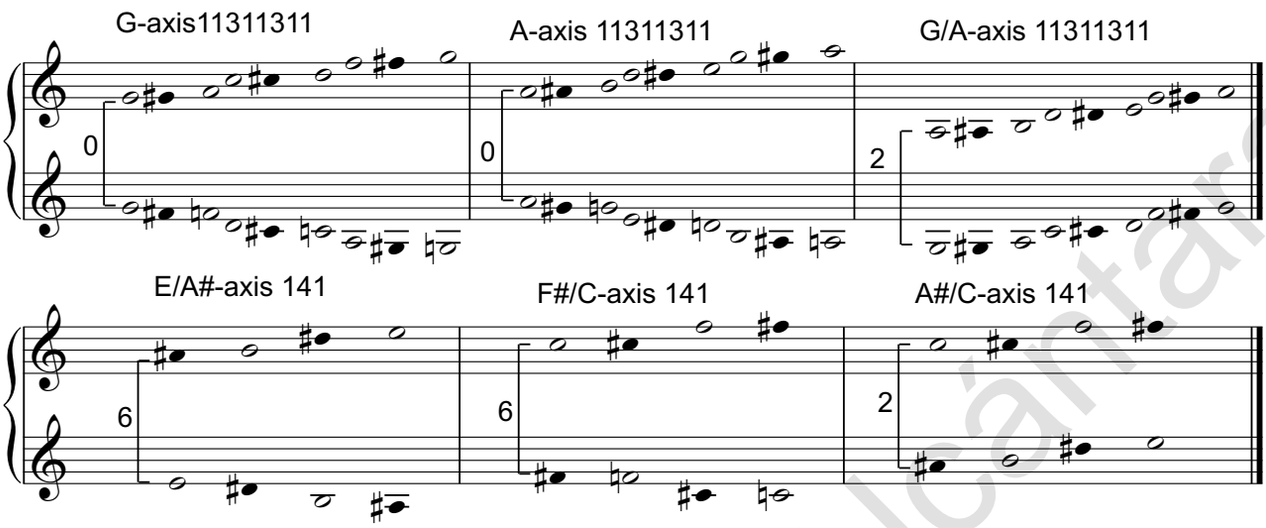
Similarly, it can be explained how the topographically inverted complement of a scale can be derived in a parallel motion. Below this is shown by using the 5th mode of G melodic minor. In bar 1, the palindromorphic primary scales of the C/F#-axis, which are topographically inverted to each other, are provided with frames. Bar 2 shows the framed extract of its complements with an enharmonic shift of the notes A# to Bb and C to B#. This results in G melodic minor in the right hand, in C# Melodic Minor in the left hand. In bar 3, the scales are completed to the octave and begin with the notes D and G#. It turns out that the topographically inverted playing of the palindromorphic scale 2212122 (parallel motion) goes back to the C/F#-axis.



© 2019 Victor Alcantara

Primary scale 11311311

Primary scales 11311311 in contrary motion (bar 1–2), combined primary scales 11311311 in parallel motion (bar 3), complements 141 in contrary motion (bar 4–5), combined complements 141 in parallel motion (bar 6)



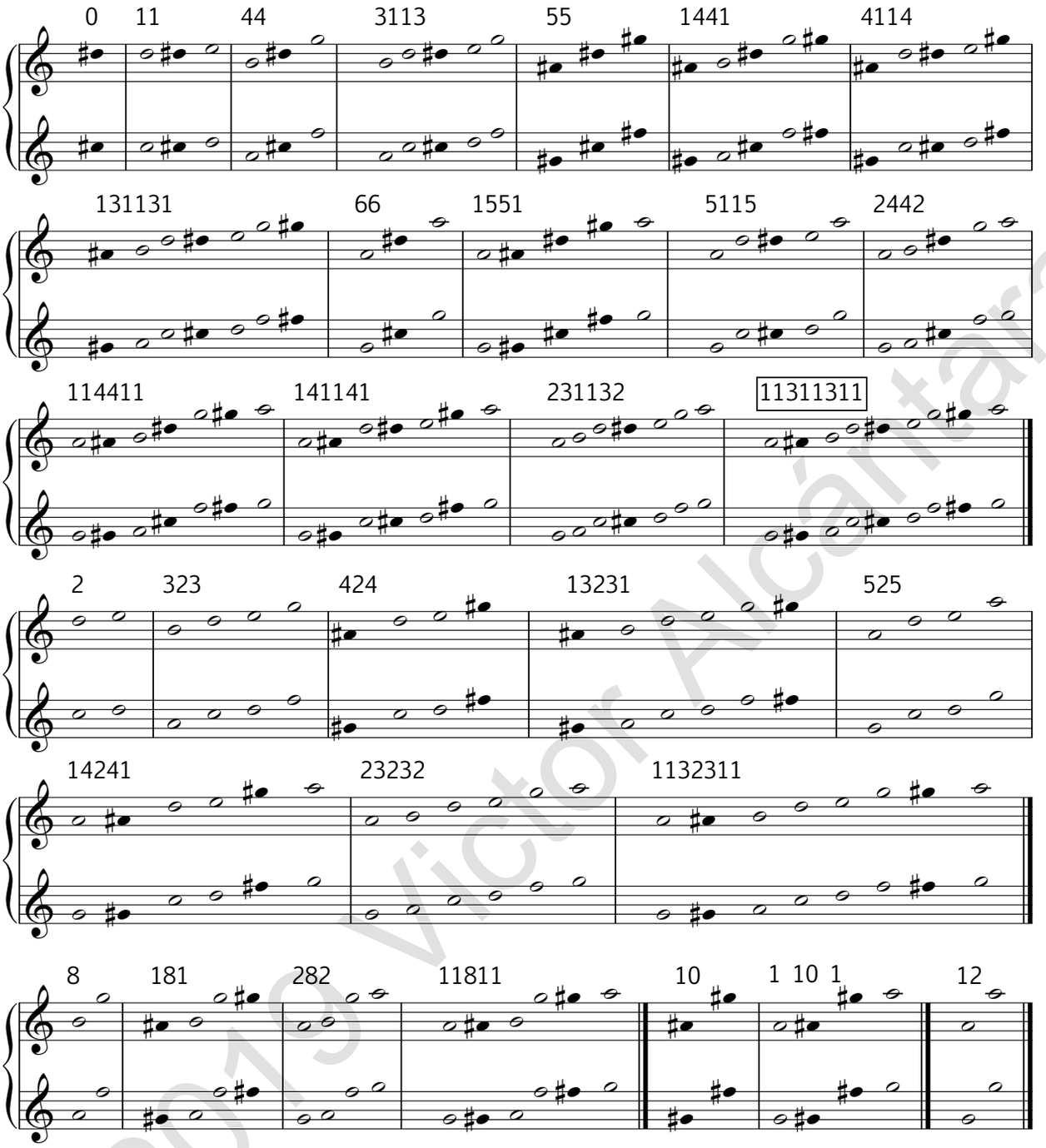
Subscales of the primary scale 11311311 and its complement 141

Primary scale 11311311 with subscales										Range	Intervals/Scale	No.
					0							
				1	0	1				2	1|0|1	1
		4			0			4		8	44	2
		3		1	0	1		3		8	3113	3
	5				0				5	10	55	4
	1	4			0			4	1	10	1441	5
	4			1	0	1			4	10	4114	6
	1	3		1	0	1		3	1	10	131131	7
6					0				6	10	66	8
1	5				0				5 1	12	1551	9
5				1	0	1			5	12	5115	10
2		4			0			4	2	12	2442	11
1	1	4			0			4	1 1	12	114411	12
1	4			1	0	1			4 1	12	141141	13
2		3		1	0	1		3	2	12	231132	14
1	**1**	**3**		**1**	**0**	**1**		**3**	**1 1**	**12**	**11311311**	**15**
					2					2	2	16
		3			2			3		8	323	17
	4				2				4	8	424	18
	1	3			2			3	1	8	13231	19
5					2				5	12	525	20
1	4				2				4 1	12	14241	21
2		3			2			3	2	12	23232	22
1	1	3			2			3	1 1	12	1132311	23
	1				8				1	10	181	24
					8							25
2					8				2	12	282	26
1	1				8				1 1	12	11811	27
					10					10	10	28
1					10				1	12	1 10 1	29
					12					12	12	30

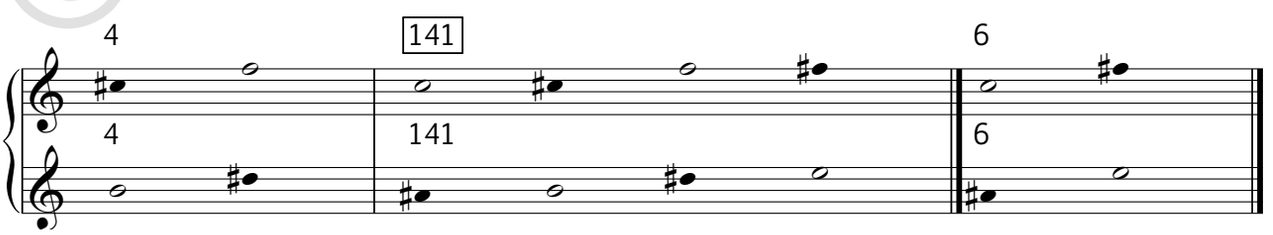
Complement 141 with subscales										Range	Intervals/Scale	No.
					4					4	4	1
		1			**4**			**1**		**6**	**141**	**2**
					6					6	6	3

© 2019 Victor Alcántara

Combined primary scales of the G/A-axis 11311311 with subscales in parallel motion



Combined complements of the A#/C-axis 141 with subscales in parallel motion



© 2019 Victor Alcántara

Final remarks

As it was shown, the structural symmetries of the keyboard may be matched with interval-scalar symmetries and thus can be experienced pianistically by means of contrary and parallel motion. The resulting multitude of palindromorphic scales can be understood on the one hand as a sort of raw material for topographically guided improvisation or composition. On the other hand, the recourse to palindromorphy in particular allows for the opposite, i.e. the creation of purposeful breaks of symmetry, e.g. with regard to counterpoint, tonality and rhythm.

In order to gain a practical insight into the creative work with palindromorphic structures, three palindromorphic original compositions are presented below. They are united under the title "Three Palindromorphic Pieces for Piano".

No. I is based mainly on Messiaen II, Messiaen III and Dorian.
No. II makes frequent use of phase-shifting and mirrored repetition.
No. III is characterized by extended sections of topographically-inverted playing.

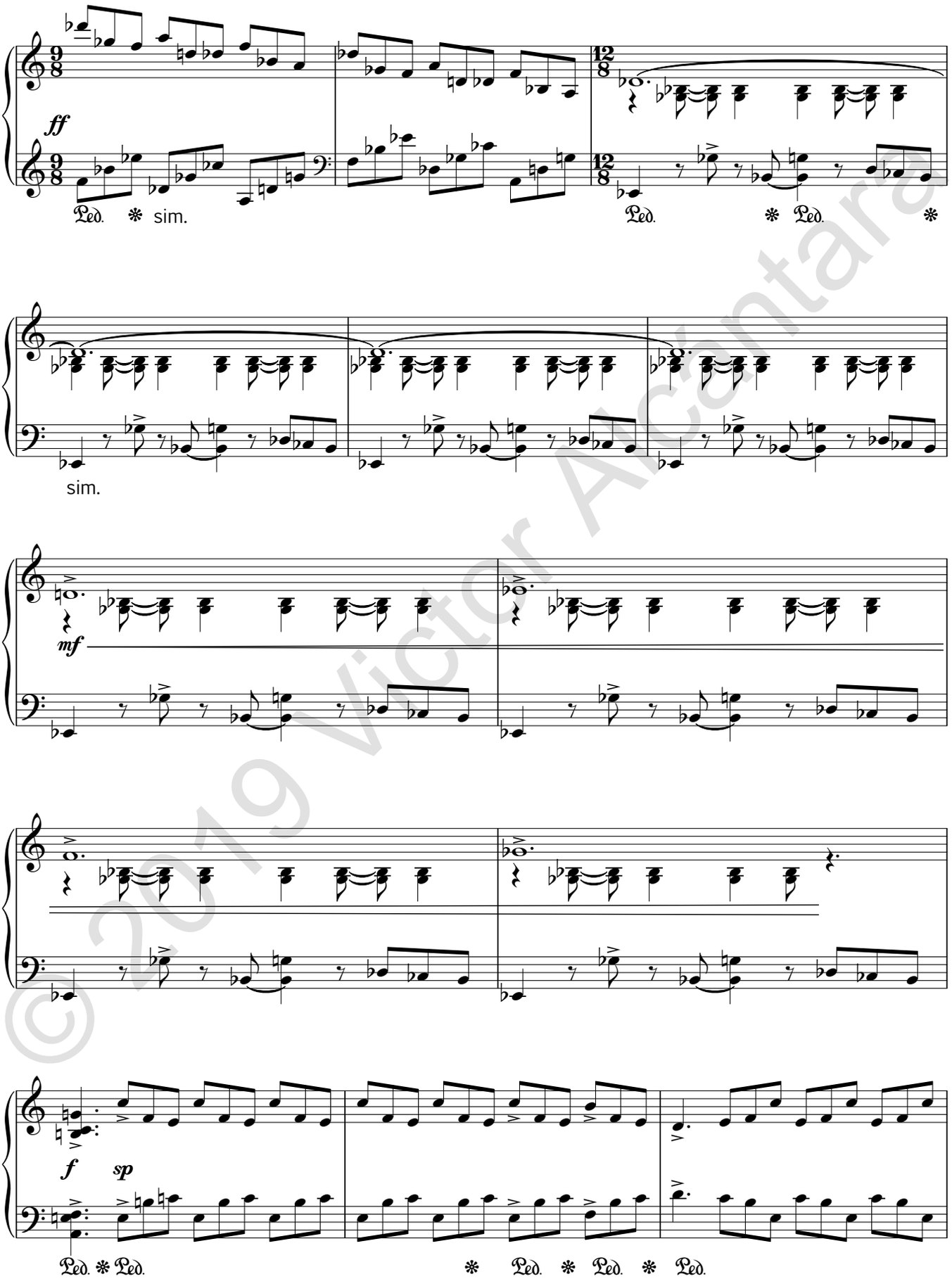
The compositions may be performed as single pieces or in the given order.

© 2019 Victor Alcántara

I.

♩.=180

Victor Alcántara (*1975)



© 2019 Victor Alcántara

Index

© 2019 Victor Alcántara

Sources

Corea, Chick
(2002) *A work in progress*, n.p. : Chick Corea Productions

Dobbins, Bill
(1984) *The contemporary jazz pianist*, Rottenburg am Neckar: Advance Music

Eschmann-Dumur, Carl
(1888) Guide de jeune pianiste, Zürich: Hug frères

Feuchtwanger, Peter
(2004) *Klavierübungen zur Heilung von physiologischen Spielstörungen und zum Erlernen eines funktional natürlichen Klavierspiels*, Stefan Blido und Manfred Seewann (Eds.), n.p.

Fuchs, Carl Dorius
(1869) *Der Grundsatz der technischen Identität symmetrischer Klavierfiguren*, in:
Berliner Musikzeitung Echo, Nr. 2, p. 5–7 of the supplement;
Virtuos und Dilettant, Stralsund: Topp

Koepf, Siegfried
(2009) *Die Skalen im Tonumfang einer Oktav*, Köln: Feedback-Studio Verlag

Messiaen, Olivier
(1944) *Technique de mon langage musical*, Paris: Leduc

Morrison, Karen
(2015) *GCSE mathematics for edexcel foundation: Student book*, Cambridge: Cambridge University Press

Norris, Walter
(2007) *Essentials for pianist improvisers*, Berlin: Sunhazed Publishing

Ovsepian, Vardan
(2011) *Mirror exercises*, n. p. : Vostevulgo Music

Rée, Anton
(1870) Beiträge zu einer systematisch-technischen Ausbildung der Hände, in: *Neue Zeitschrift für Musik*, Nr. 11, p. 101–104

Rossomandi, Florestan
(1923) *Guida per lo studio tecnico del pianoforte*, Neapel: Salvatore Simeoli

Södling, Carl Erik
(1860) *Nytt system för piano-undervisning*, Stockholm: Beckman

Teichmüller, Robert
(1937) *Klavier-Technik*, Leipzig: Franz Jost

Verbalis, John
(2012) *Natural fingering: a topographical approach to pianism*, New York, NY: Oxford University Press

Ziehn, Bernhard
(1881) *System der Übungen*, Hamburg: Pohle

© 2019 Victor Alcántara